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# Integral representation of holomorphic mappings on fully nuclear spaces ☆

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**Abstract**

We obtain the following integral representation:

$$f(z)=\int\limits_{E'_\beta} e^{\langle z,w\rangle}\cdot(f\circ D)(w)\,d\mu_\gamma(w)$$

for all $z\in E$, a fully nuclear space with basis, where $\eta$ and $\gamma$ belong to $E'_\beta$, $\eta/\gamma\in\ell_1$, $f$ is a holomorphic function of $\eta$-exponential type on $E$, $\mu_\gamma$ is a Gaussian measure on $E'_\beta$, and $D$ is a densely defined diagonal mapping from $E'_\beta$ into $E$.


*Keywords:* Gaussian measure; Nuclear space; Holomorphic function of exponential type

## 1. Introduction

This paper is motivated by results by Pinasco and Zalduendo in [3,4] on the integral representation of holomorphic functions defined on Banach spaces. We follow the same basic approach by using Gaussian measures in considering holomorphic functions on fully nuclear spaces with a basis. In this setting we have absolutely convergent monomial expansions for holomorphic mappings and Minlos's Fourier transform characterisation of Gaussian measures [1]. The authors thank Nacho Zalduendo for introducing them to the subject and for his extremely helpful advice.

A fully nuclear space with basis is a complete barrelled nuclear space $E$ with basis $(e_n)_{n=1}^\infty$ such that its strong dual is also a complete barrelled nuclear space with basis. A fully nuclear space with basis is reflexive and its strong dual $E'_\beta$ is also fully nuclear with absolute basis $(e'_n)_{n=1}^\infty$. Fréchet nuclear spaces with basis and their strong duals are

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fully nuclear with basis. Moreover, the collection of fully nuclear spaces with basis is closed with respect to countable products and direct sums. We let

$$\left\langle \sum_{n=1}^{\infty} z_n e_n, \sum_{n=1}^{\infty} w_n e_n' \right\rangle = \sum_{n=1}^{\infty} z_n \bar{w}_n$$

denote the dual pairing between $E$ and $E'_\beta$. We may also use this dual pairing to define a fundamental system of semi-norms on $E$; if $\sum_{n=1}^{\infty} \beta_n e_n' \in E'_\beta$, then $\sum_{n=1}^{\infty} z_n e_n \in E \to \sum_{n=1}^{\infty} |\beta_n z_n|$ defines a continuous semi-norm on $E$ and if we consider all such semi-norms we obtain a fundamental system of semi-norms. We shall always suppose that $E$ is a complex space and denote by $E_{\mathbb{R}}$ the space $E$ with its underlying real structure. If we let $e_{n*} = ie_n$, where $i = \sqrt{-1}$, then it is easily seen that $(e_n, e_{m*})_{n,m=1}^{\infty}$ is an absolute basis for $E_{\mathbb{R}}$. Moreover, the complexification of $E_{\mathbb{R}}$ is isomorphic to $E \times E$. If $\sum_{n=1}^{\infty} z_n e_n \in E$, then there exists $\sum_{n=1}^{\infty} z_n' e_n \in E$ such that $(|z_n|/|z_n'|)_{n=1}^{\infty} \in \ell_1$.[1]

By [2] the monomials form an absolute basis for $\mathcal{H}(E)$, the entire functions on $E$, with respect to any of the usual topologies, including the compact open topology $\tau_0$, whenever $E$ is a fully nuclear space with basis. If $\theta := (\theta_n)_{n=1}^{\infty}$ and $z := (z_n)_{n=1}^{\infty}$ are sequences of complex numbers we let

$$\|z\|_\theta = \sum_{n=1}^{\infty} |z_n| \cdot |\theta_n| = \|\theta\|_z.$$

We refer to [2] for further details on fully nuclear spaces with basis and the theory of holomorphic functions on these spaces.

A holomorphic mapping $f \in \mathcal{H}(E)$ which satisfies any of the equivalent conditions in the following proposition is said to be of *$\eta$-exponential type*.

**Proposition 1.** *If $E$ is a fully nuclear space with basis, $\eta_n \geqslant 0$ for all $n$, and $\sum_{n=1}^{\infty} \eta_n e_n' \in E'_\beta$, then the following are equivalent conditions on $f \in \mathcal{H}(E)$:*

(a) *$f(z) := \sum_{m \in \mathbb{N}^{(\mathbb{N})}} a_m z^m \in \mathcal{H}(E)$ satisfies $|a_m| \leqslant ab^{|m|} \eta^m / m!$ for some $a, b > 0$ and all $m \in \mathbb{N}^{(\mathbb{N})}$,*
(b) *$f = \sum_{n=0}^{\infty} P_n$, where $P_n$ is a continuous $n$-homogeneous polynomial for each $n$ and $|P_n(z)| \leqslant cd^n \|z\|_\eta^n / n!$ for all $n$ and all $z \in E$ and some $c, d > 0$,*
(c) *$|f(z)| \leqslant Ae^{B\|z\|_\eta}$ for all $z \in E$, where $A$, $B$ are positive constants.*

**Proof.** If (a) holds, then for all $n$,

$$\begin{aligned} \left|P_n(z)\right| &\leqslant \sum_{m \in \mathbb{N}^{(\mathbb{N})},\, |m|=n} |a_m| \cdot \left|z^m\right| \\ &\leqslant ab^{|m|} \sum_{m \in \mathbb{N}^{(\mathbb{N})},\, |m|=n} \eta^m \left|z^m\right| / m! \\ &= ab^n \|z\|_\eta^n / n! \end{aligned}$$

and (a) implies (b). If (b) holds, then

$$\left|f(z)\right| \leqslant \sum_{n=0}^{\infty} \left|P_n(z)\right| \leqslant c \sum_{n=0}^{\infty} \frac{d^n \|z\|_\eta^n}{n!} \leqslant ce^{d\|z\|_\eta}$$

and (b) implies (c).

Suppose (c) holds. For any sequence of positive scalars $(\alpha_i)_{i=1}^{\infty}$ the set $D := \{(z_i)_{i=1}^n\colon\ |z_i| = \alpha_i/\eta_i,\ i = 1, \ldots, n\}$ is the distinguished boundary of the compact polydisc $\{(z_i)_{i=1}^n\colon\ |z_i| \leqslant \alpha_i/\eta_i,\ i = 1, \ldots, n\}$ and

$$\sup\left\{\|z\|_\eta\colon\ z \in D\right\} = \sup\left\{ \sum_{i=1}^{n} |z_i|\eta_i\colon\ |z_i| = \alpha_i/\eta_i \right\} = \sum_{i=1}^{n} \alpha_i.$$

[1] For convenience we use the notation $\frac{0}{0} = 0$.

If $m = (m_1, \dots, m_n) \in \mathbb{N}^{(\mathbb{N})}$, then, using Cauchy estimates, we have

$$\frac{\alpha^m}{\eta^m}|a_m| = \left\|a_m z^m\right\|_{\{z\in D\}} \leqslant \left\|f(z)\right\|_{\{z\in D\}} \leqslant Ae^{B\sup\{\|z\|_\eta:\ z\in D\}} \leqslant Ae^{B\sum_{i=1}^n \alpha_i}.$$

Hence

$$|a_m| \leqslant A\eta^m \inf_{\alpha_i>0}\left(\prod_{1\leqslant i\leqslant n}\frac{e^{B\alpha_i}}{\alpha_i^{m_i}}\right)$$

and on letting $\alpha_i = m_i/B$ we obtain

$$|a_m| \leqslant A\eta^m \prod_{1\leqslant i\leqslant n}\frac{(Be)^{m_i}}{m_i^{m_i}} \leqslant A\eta^m \prod_{1\leqslant i\leqslant n}\frac{(Be)^{m_i}}{m_i!} \leqslant A(Be)^{|m|}\eta^m/m!.$$

Hence (c) implies (a) and this completes the proof. □

Let $E$ denote a fully nuclear space with basis over $\mathbb{C}$. We identify, $we_j' := (x+iy)e_j' \in E_\beta'$ with $xe_j' + ye_{j*}' \in (E_\beta')_\mathbb{R}$ and $ze_j := (u+iv)e_j \in E$ with $ue_j + v_je_{j*} \in E_\mathbb{R}$ for $x, y, u$ and $v$ in $\mathbb{R}$. If $Q : E_\mathbb{R} \to \mathbb{R}$ is a positive, that is $Q(x) \geqslant 0$ for all $x \in E_\mathbb{R}$, continuous quadratic form, then by a result of Minlos, see [1, Proposition 4, p. 76, and Corollaire, p. 92], there is a Gaussian probability measure $\mu$ on $E_\beta'$ such that for all $z \in E$,

$$\left[\mathcal{F}(\mu)\right](z) := \int_{E_\beta'} e^{i\langle z,w\rangle}\,d\mu(w) = e^{-Q(z)/2}.$$

The mapping $\mathcal{F}(\mu)$ is called the Fourier transform of $\mu$.

**Lemma 1.** *If $(e_n)_{n=1}^\infty$ is an absolute basis for the fully nuclear space $E$ and $(\gamma_n)_{n=1}^\infty$ is a sequence of non-negative scalars such that $\sum_{n=1}^\infty \gamma_n e_n' \in E_\beta'$, then $Q(\sum_{n=1}^\infty x_ne_n + \sum_{m=1}^\infty y_me_{m*}) = \sum_{n=1}^\infty \gamma_n^2(x_n^2+y_n^2)$ defines a continuous positive quadratic form on $E_\mathbb{R}$. Moreover, if $\mu_\gamma$ is the Gaussian measure on $(E_\mathbb{R})_\beta'$ such that $\mathcal{F}(\mu_\gamma) = e^{-Q/2}$, then for any cylindrical Borel set $B$ with base in the subspace of $(E_\mathbb{R})_\beta'$ spanned by $(e_n', e_{m*}')_{n,m=1}^k$ we have*

$$\begin{aligned}\mu_\gamma(B) &= \frac{1}{\pi^k(\gamma_1\cdots\gamma_k)^2}\int_B e^{-\sum_{n=1}^k \frac{x_n^2+y_n^2}{\gamma_n^2}}\,dx_1\,dy_1\cdots dx_k\,dy_k \\ &= \frac{i^k}{(2\pi)^k(\gamma_1\cdots\gamma_k)^2}\int_B e^{-\sum_{n=1}^k \frac{|w_n|^2}{\gamma_n^2}}\,dw_1\,d\bar{w}_1\cdots dw_k\,d\bar{w}_k.\end{aligned}$$

**Proof.** Since $\gamma_n \geqslant 0$ for all $n$, $Q$ is a positive quadratic form. If $\sum_{n=1}^\infty \gamma_ne_n' \in E_\beta'$, then, by nuclearity, we can find a sequence of positive real numbers $(\delta_n)_{n=1}^\infty$ such that $\sum_{n=1}^\infty 1/\delta_n < \infty$ and $\sum_{n=1}^\infty \delta_n\gamma_ne_n' \in E_\beta'$. By duality $V := \{\sum_{n=1}^\infty z_ne_n \in E\colon \sup_n|\delta_n\gamma_nz_n| \leqslant 1\}$ is a neighbourhood of zero in $E$. Since

$$\sum_{n=1}^\infty \sup\left\{\gamma_n^2\left(x_n^2+y_n^2\right):\ \sum_{n=1}^\infty x_ne_n \in kV,\ \sum_{m=1}^\infty y_me_{m*} \in kV\right\} \leqslant 2k^2\sum_{n=1}^\infty \delta_n^{-2} < \infty$$

for all positive integers $k$, $Q$ is the limit, uniformly over $kV$, of the sequence of continuous positive quadratic forms

$$Q_l\left(\sum_{n=1}^\infty x_ne_n + \sum_{m=1}^\infty y_me_{m*}\right) := \sum_{n=1}^l \gamma_n^2\left(x_n^2+y_n^2\right).$$

Hence $Q$ is continuous and by the result of Minlos, quoted above, there exists a Gaussian probability measure $\mu$ such that $\mathcal{F}(\mu) = e^{-Q/2}$. The proof is completed by noting that $dx\,dy = i\,dz\,d\bar{z}/2$. □

With the above notation we have the following result.

**Lemma 2.** *Let $\sum_{n=1}^{\infty}\gamma_n e'_n \in E'_\beta$, where $\gamma_n \geqslant 0$ for all $n$. Let $(\theta_n)_{n=1}^{\infty}$ denote a sequence of real numbers and suppose $1 \leqslant p < \infty$. If $\sum_{n=1}^{\infty}|\theta_n|\gamma_n < \infty$, then the mapping $\sum_{n=1}^{\infty} w_n e'_n \in E'_\beta \to e^{\|w\|_\theta}$ belongs to $\mathcal{L}^p(\mu_\gamma)$.*

**Proof.** We have $|e^{\|w\|_\theta}|^p = e^{p\sum_{n=1}^{\infty}|\theta_n|\cdot|w_n|}$. If $g_k : w = \sum_{n=1}^{\infty} w_n e'_n \in E'_\beta \to e^{p\sum_{n=1}^{k}|\theta_n|\cdot|w_n|}$, then the sequence $(g_k)_{k=1}^{\infty}$ is increasing and it suffices by the Monotone Convergence Theorem to show $\lim_{k\to\infty}\int_{E'_\beta} g_k\,d\mu_\gamma$ is finite. On letting $q = p/2$ we have

$$\begin{aligned}
\int_{E'_\beta} g_k(w)\,d\mu_\gamma(w) &= \frac{i^k}{(2\pi)^k(\gamma_1\cdots\gamma_k)^2}\int_{\mathbb{C}^k} e^{p\sum_{n=1}^{k}|\theta_n|\cdot|w_n|} e^{-\sum_{n=1}^{k}\frac{|w_n|^2}{\gamma_n^2}}\,dw_1\,d\bar{w}_1\cdots dw_k\,d\bar{w}_k \\
&= \prod_{n=1}^{k}\frac{i}{2\pi\gamma_n^2}\int_{\mathbb{C}} e^{p|\theta_n|\cdot|w_n| - \frac{|w_n|^2}{\gamma_n^2}}\,dw_n\,d\bar{w}_n \\
&= \prod_{n=1}^{k}\frac{i e^{q^2|\theta_n|^2\gamma_n^2}}{2\pi\gamma_n^2}\int_{\mathbb{C}} e^{-\frac{1}{\gamma_n^2}(|w_n| - q|\theta_n|\gamma_n^2)^2}\,dw_n\,d\bar{w}_n \\
&= e^{\sum_{n=1}^{k} q^2|\theta_n|^2\gamma_n^2}\prod_{n=1}^{k}\frac{1}{\gamma_n^2}\int_0^\infty 2s e^{-\frac{1}{\gamma_n^2}(s - q|\theta_n|\gamma_n^2)^2}\,ds \\
&= e^{\sum_{n=1}^{k} q^2|\theta_n|^2\gamma_n^2}\prod_{n=1}^{k}\int_0^\infty 2r e^{-(r - q|\theta_n|\gamma_n)^2}\,dr.
\end{aligned}$$

We now consider the integral

$$f(\alpha) := \int_0^\infty 2r e^{-(r-\alpha)^2}\,dr$$

as a function of $\alpha \geqslant 0$ on the interval $[0, 1]$. We have $f(0) = 1$ and, by the Mean Value Theorem, for $0 < \alpha < 1$ and $r > 0$,

$$\left|\frac{e^{-(r-\alpha)^2} - e^{-r^2}}{(r-\alpha) - r}\right| = 2r_\alpha e^{-r_\alpha^2},$$

where $r - \alpha < r_\alpha < r$. Hence

$$\left|e^{-(r-\alpha)^2} - e^{-r^2}\right| \leqslant 2\alpha g(r) := \begin{cases} 2\alpha & \text{if } 0 \leqslant r \leqslant 1, \\ 2\alpha r e^{-(r-1)^2} & \text{if } r > 1, \end{cases}$$

and

$$\left|f(\alpha) - f(0)\right| \leqslant \int_0^\infty 2r\left|e^{-(r-\alpha)^2} - e^{-r^2}\right|dr \leqslant 4\alpha\int_0^\infty r g(r)\,dr.$$

As the function $r \to rg(r)$ is integrable, the constant $C := 4\int_0^\infty rg(r)\,dr$ is finite and $|f(\alpha) - f(0)| \leqslant C\alpha$ for $0 \leqslant \alpha < 1$. This implies

$$\sum_{n=1}^{\infty}\left|f\big(q|\theta_n|\gamma_n\big) - f(0)\right| \leqslant Cq\sum_{n=1}^{\infty}|\theta_n|\gamma_n < \infty$$

and hence $\prod_{n=1}^{\infty}(1 + (f(q|\theta_n|\gamma_n) - f(0))) < \infty$. We now have

$$\begin{aligned}\lim_{k\to\infty}\int\limits_{E'_\beta} g_k\,d\mu_\gamma &= e^{\sum_{n=1}^\infty q^2|\theta_n|^2\gamma_n^2}\prod_{n=1}^\infty\int\limits_0^\infty 2re^{-(r-q|\theta_n|\gamma_n)^2}\,dr\\ &\leqslant e^{\sum_{n=1}^\infty q^2|\theta_n|^2\gamma_n^2}\prod_{n=1}^\infty f\big(q|\theta_n|\gamma_n\big)\\ &<\infty.\end{aligned}$$

This completes the proof. □

We shall also need the following result (see [4, Lemma 2.1]).

**Lemma 3.**

$$\int\limits_{E'_\beta} w^m\bar{w}^{m'}\,d\mu_\gamma = \begin{cases} m!\gamma^{2m} & \textit{if } m=m',\\ 0 & \textit{otherwise}.\end{cases}$$

**Proof.** If $m=(m_1,\dots,m_k)$ and $m'=(m'_1,\dots,m'_k)$, then

$$\begin{aligned}\int\limits_{E'_\beta} w^m\bar{w}^{m'}\,d\mu_\gamma &= \frac{i^k}{(2\pi)^k(\gamma_1\cdots\gamma_k)^2}\int\limits_{\mathbb{C}^k} w^m\bar{w}^{m'}e^{-\sum_{n=1}^k\frac{|w_n|^2}{\gamma_n^2}}\,dw_1\,d\bar{w}_1\cdots dw_k\,d\bar{w}_k\\ &=\prod_{n=1}^k\frac{i}{2\pi\gamma_n^2}\int\limits_{\mathbb{C}} w^{m_n}\bar{w}^{m'_n}e^{-\frac{|w|^2}{\gamma_n^2}}\,dw\,d\bar{w}\end{aligned}$$

and this is 0 if $m\neq m'$. If $m=m'$, then

$$\begin{aligned}\int\limits_{E'_\beta} w^m\bar{w}^{m'}\,d\mu_\gamma &= \prod_{n=1}^k\frac{i}{2\pi\gamma_n^2}\int\limits_{\mathbb{C}}|w|^{2m_n}e^{-\frac{|w|^2}{\gamma_n^2}}\,dw\,d\bar{w}\\ &=\prod_{n=1}^k\frac{1}{\pi\gamma_n^2}\int\limits_{\mathbb{R}^2}\big(x^2+y^2\big)^{m_n}e^{-\frac{x^2+y^2}{\gamma_n^2}}\,dx\,dy\\ &=\prod_{n=1}^k\frac{2}{\gamma_n^2}\int\limits_0^\infty r^{2m_n}e^{-\frac{r^2}{\gamma_n^2}}r\,dr\\ &=\prod_{n=1}^k\gamma^{2m_n}\int\limits_0^\infty u^{m_n}e^{-u}\,du\\ &=\gamma^{2m}m!.\qquad\square\end{aligned}$$

The following is the main result in this paper.

**Proposition 2.** *Let $E$ denote a fully nuclear space with basis $(e_n)_{n=1}^\infty$, strong dual $E'$ and dual basis $(e'_n)_{n=1}^\infty$. Let $\sum_{n=1}^\infty\eta_ne'_n$ belong to $E'_\beta$, $\eta_n\geqslant 0$ for all $n$, and suppose $f\in\mathcal{H}(E)$ is of $\eta$-exponential type. Let $\sum_{n=1}^\infty\gamma_ne'_n\in E'_\beta$, $\gamma_n\geqslant 0$ all $n$, be chosen so that $\sum_{n=1}^\infty\eta_n/\gamma_n<\infty$. If $D:E'_\beta\to E$ denotes the densely defined linear operator $D(\sum_{n=1}^\infty w_ne'_n):=\sum_{n=1}^\infty\gamma_n^{-2}w_ne_n$ and $\mu_\gamma$ denotes the measure on $E'_\beta$ that we have previously associated with $\gamma$, then*

$$f(z) = \int_{E'_\beta} e^{\langle z,w\rangle} \cdot (f \circ D)(w)\, d\mu_\gamma(w) \tag{1}$$

*for all $z \in E$.*

**Proof.** Let $f(z) = \sum_{m\in\mathbb{N}^{(\mathbb{N})}} a_m z^m$ for all $z \in E$. By our hypothesis there exist $a > 0$ and $b > 0$ such that $|a_m| \leqslant ab^{|m|}\eta^m/m!$ for all $m \in \mathbb{N}^{(\mathbb{N})}$. Since

$$\begin{aligned}\sum_{m\in\mathbb{N}^{(\mathbb{N})}} |a_m| \cdot \left|\left(D(w)\right)^m\right| &\leqslant a \sum_{m\in\mathbb{N}^{(\mathbb{N})}} b^{|m|}\eta^m\left(|w|\gamma^{-2}\right)^m/m! \\ &= a \sum_{m\in\mathbb{N}^{(\mathbb{N})}} \frac{b^{|m|}(|w|\eta\gamma^{-2})^m}{m!} \\ &= ae^{b\|w\|_{\eta\gamma^{-2}}}\end{aligned} \tag{2}$$

and

$$\sum_{n=1}^{\infty}\left(\frac{\eta_n}{\gamma_n^2}\right)\gamma_n = \sum_{n=1}^{\infty}\frac{\eta_n}{\gamma_n} < \infty.$$

Lemma 2 implies that the mapping

$$f \circ D : w \in E'_\beta \to \sum_{m\in\mathbb{N}^{(\mathbb{N})}} a_m \cdot \left(D(w)\right)^m$$

converges pointwise to an element in $\mathcal{L}^p(\mu_\gamma)$ for all $p \geqslant 1$.

If $|z| := \sum_{n=1}^{\infty} |z_n| e_n \in E$ and $w := \sum_{n=1}^{\infty} w_n e'_n \in E'_\beta$, then

$$\left|e^{\sum_{n=1}^{\infty} z_n\cdot\bar{w}_n}\right| \leqslant e^{\sum_{n=1}^{\infty}|z_n|\cdot|w_n|} = \sum_{m\in\mathbb{N}^{(\mathbb{N})}} \frac{|z|^m \cdot |w|^m}{m!} = e^{\|w\|_{|z|}} \tag{3}$$

and, since $\sum_{n=1}^{\infty}|z_n|\cdot\gamma_n < \infty$, Lemma 2 implies that the mapping $w \in E'_\beta \to e^{\sum_{n=1}^{\infty}|z_n||\bar{w}_n|}$ belongs to $\mathcal{L}^p(\mu_\gamma)$ for all $p$, $1 \leqslant p < \infty$.

By Hölder's inequality the mapping

$$w \in E'_\beta \to e^{\langle z,w\rangle} \cdot (f \circ D)(w)$$

is integrable and, by (2) and (3), we may integrate term by term to obtain, using Lemma 3,

$$\begin{aligned}\int_{E'_\beta} e^{\langle z,w\rangle} \cdot (f\circ D)(w)\, d\mu_\gamma(w) &= \int_{E'_\beta} \sum_{m',m\in\mathbb{N}^{(\mathbb{N})}} \frac{z^{m'}\bar{w}^{m'}}{m'!} a_m\left(D(w)\right)^m d\mu_\gamma \\ &= \sum_{m',m\in\mathbb{N}^{(\mathbb{N})}} \frac{a_m z^{m'}}{m'!}\cdot\gamma^{-2m}\int_{E'_\beta} w^m\bar{w}^{m'}\, d\mu_\gamma \\ &= \sum_{m\in\mathbb{N}^{(\mathbb{N})}} \frac{a_m z^m}{m!}\cdot\gamma^{-2m} m!\gamma^{2m} \\ &= \sum_{m\in\mathbb{N}^{(\mathbb{N})}} a_m z^m \\ &= f(z).\end{aligned}$$

This proves (1) and completes the proof. □

## References

[1] N. Bourbaki, Éléments de Mathématique, Intégration IX, Act. Sc. et Ind., vol. 1343, Hermann, Paris, 1969.
[2] S. Dineen, Complex Analysis on Infinite Dimensional Spaces, Monogr. Math., Springer-Verlag, 1999.
[3] D. Pinasco, Integral representations of holomorphic functions on Banach spaces II, preprint.
[4] D. Pinasco, I. Zalduendo, Integral representations of holomorphic functions on Banach spaces, J. Math. Anal. Appl. 308 (2005) 159–174.